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NONSTEADY FLOWS OF VISCOPLASTIC FLUIDS AT THE INITIAL SECTIONS OF PLANE CHANNELS

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The problem is formulated and the method of solving internal problems of rheodynamics of nonsteady flows of viscoplastic fluids is proposed.

Nonsteady motions of viscoplastic media are of considerable interest in connection with investigations of technological processes occurring under dynamic loading. A number of articles are devoted to an analysis of the rheodynamics of nonsteady flows on spatially steady sections, a review of which is given in [1]. The initial sections for steady flow of viscoplastic fluids were considered in [2-5]. Investigations of nonsteady flows at initial sections of channels have so far not been carried out.

We consider the flow of a fluid in a plane channel (Fig. 1). The velocity of the fluid at the inlet is constant over the cross section and equal to V. From physical considerations the entire flow region can be divided into two regions: a zone of shearing flow, adjoining the wall of the channel ($\delta < y \leq h$), and a "quasisolid" core, where the velocity is constant ($0 \leq y \leq \delta$). We should note that the velocity of the quasisolid core U(x) varies along the channel axis. Under such conditions we can use the modified model of a viscoplastic fluid, which for $\tau \leq \tau_0$ exhibits creep, i.e., slow flow with high viscosity. Then, the nonuniqueness of the velocity field in the transverse direction can be neglected; in this case, however, the model admits the dependence of U on the longitudinal coordinate.

The equations of motion in the boundary-layer approximation are

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}\right)=-\dfrac{\partial p}{\partial x}+\mu\dfrac{\partial^2 u}{\partial y^2}, & (1) \\ \qquad\qquad (\delta<y\leqslant h) & \\ \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0, & (2) \end{cases}$$

$$\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\right)=-\frac{\partial p}{\partial x}-\frac{\tau_0}{\delta}\quad(0\leqslant y\leqslant\delta) \tag{3}$$

(an investigation was carried out for the linear model of a viscoplastic Shvedov–Bingham medium).

Equation (3) for a quasisolid core contributes the term τ_0/δ. It indicates that the stresses on the boundary of the quasisolid core are equal to τ_0. In [2] for the steady-state problem in the zone of quasisolid motion, the Shiller approximation was assumed to be valid:

$$U\frac{\partial U}{dx}=-\frac{1}{\rho}\frac{dp}{dx}. \tag{4}$$

In our opinion, neglecting the term τ_0/δ can lead to sizable errors in the solution.

Let the medium be at rest for $t \leq 0$, and for $t > 0$ let there be a flow with a constant flow rate. The initial and boundary conditions of the problem are the following:

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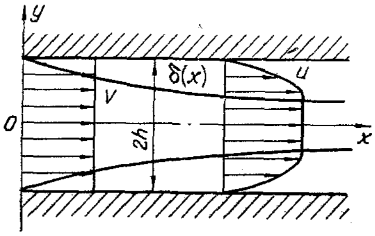

Fig. 1. Flow diagram.

$$u(0,\ x,\ y)=0,\quad u(t,\ x,\ h)=0,$$

$$u(t,\ 0,\ y)=V,\quad \frac{\partial u(t,\ x,\ \delta)}{\partial y}=0 \qquad (5)$$
$$u(t,\ x,\ \delta)=U.$$

Furthermore, the following integral condition is satisfied:

$$\rho\int_{0}^{h} u\,dy=Q. \qquad (6)$$

Introduction of the dimensionless variables and parameters

$$t'=\frac{tV}{h},\quad \xi=\frac{x}{h},\quad \eta=\frac{y}{h},\quad \delta'=\frac{\delta}{h},$$
$$w=\frac{u}{V},\quad \varphi=\frac{v}{V},\quad W'=\frac{U}{V},\quad s=\frac{\tau_0}{\rho V^2}, \qquad (7)$$
$$\mathrm{Re}=\frac{\rho Vh}{\mu},\quad p'=\frac{p}{\rho V^2},$$

where $V = Q/\rho h$, leads to the problem (4)-(6) after algebraic transformations to (the primes are dropped)

$$\frac{\partial w}{\partial t}+\frac{\partial}{\partial\xi}w^2+\frac{\partial}{\partial\eta}(w\varphi)=-\frac{\partial p}{\partial\xi}+\frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial\eta^2},$$
$$\frac{\partial w}{\partial\xi}+\frac{\partial\varphi}{\partial\eta}=0\quad(\delta<\eta\leqslant 1), \qquad (8)$$
$$\frac{\partial W}{\partial t}+W\frac{\partial W}{\partial\xi}=-\frac{\partial p}{\partial\xi}-\frac{s}{\delta}\quad(0\leqslant\eta\leqslant\delta),$$

$$w(0,\ \xi,\ \eta)=0,\quad \frac{\partial w(t,\ \xi,\ \delta)}{\partial\eta}=0, \qquad (9)$$

$$w(t,\ 0,\ \eta)=1,\quad w(t,\ \xi,\ 1)=0,$$
$$w(t,\ \xi,\ \delta)=W(t,\ \xi),$$

$$\int_{0}^{1} w\,d\eta=1. \qquad (10)$$

There are serious mathematical difficulties associated with the exact solution of the problem (8)-(10). Therefore, in the present work we apply the approximate integral method. The velocity profile in the boundary layer is approximated by a second-order parabola, which, with account of the constancy of the velocity in the quasisolid core, yields

$$w(t,\ \xi,\ \eta)=\begin{cases} W(t,\ \xi)\left[1-\left(\dfrac{\eta-\delta}{1-\delta}\right)^2\right] & (\delta<\eta\leqslant 1),\\ W(t,\ \xi) & (0\leqslant\eta\leqslant\delta).\end{cases} \qquad (11)$$

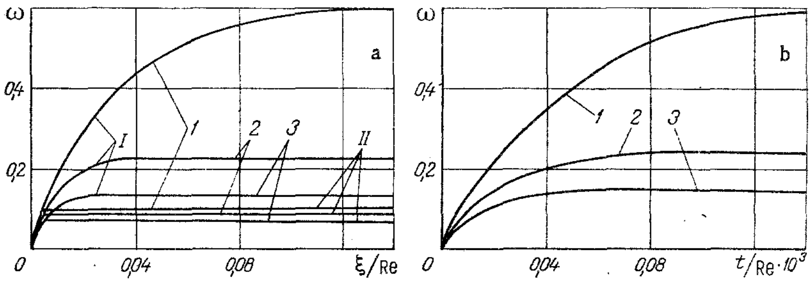

Fig. 2. Dependence of the quantity ω on the longitudinal coordinate in the steady-state regime (I) and for $t/Re = 10^{-5}$ (II) (a); change in ω in the nonsteady regime (b): 1) s = 1; 2) 5; 3) 10.

Integration of the first equation of system (8) across the boundary layer with account of relation (11) and elimination of the pressure leads to

$$\frac{1-\delta}{3}\frac{\partial W}{\partial t}-\frac{W}{3}\frac{\partial\delta}{\partial t}+\frac{4}{15}W(1-\delta)\frac{\partial W}{\partial\xi}-\frac{2}{15}W^2\frac{\partial\delta}{\partial\xi}+\frac{\partial W}{\partial\xi}\frac{1-\delta}{3}W=-\frac{s(1-\delta)}{\delta}+\frac{2W}{(1-\delta)\,\mathrm{Re}}. \quad (12)$$

The integral balance of mass (9) enables us to formulate the relation between the velocity of the quasi-solid core and the width of the shear zone

$$W=\frac{3}{2+\delta}. \quad (13)$$

We use Eq. (13) to eliminate the velocity W from (12), arriving at a first-order differential equation in the coordinate of the boundary of the zone of shear flow

$$\frac{3}{(2+\delta)^2}\frac{\partial\delta}{\partial t}+\frac{39-21\delta}{5(2+\delta)^3}\frac{\partial\delta}{\partial\xi}=\frac{6}{(1-\delta)(2+\delta)\,\mathrm{Re}}-\frac{s(1-\delta)}{\delta}. \quad (14)$$

From physical considerations the initial conditions

$$\delta(0,\ \xi)=0,\quad \delta(t,\ 0)=0 \quad (15)$$

follow.

There is interest in analyzing the limiting case of the problem – flow of a Newtonian fluid stabilized with respect to ξ and t.

In this case the Shiller approximation (4) gives sizable errors only near the inlet to the channel, i.e., for small ξ. In the region of stabilized flow, the errors in the solution are very great, since Eq. (4), in the limiting case for $\xi \gg 1$, gives a zero value for the pressure gradient. In the given scheme we use a modification of the Shiller formulation. For steady flow of a viscoplastic fluid, from Eq. (14) it follows that

$$\frac{6}{(1-\delta)(2+\delta)\,\mathrm{Re}}=\frac{s(1-\delta)}{\delta}. \quad (16)$$

For small s the following approximation is valid:

$$\delta = As. \quad (17)$$

From (16) and (17), by neglecting terms with high orders of smallness it follows that

$$A=\mathrm{Re}/3. \quad (18)$$

From Eqs. (17) and (18) and the third equation of system (8) it follows that for the region of stabilized flow for small s

$$-\frac{dp}{d\xi}=\frac{3}{\mathrm{Re}}. \quad (19)$$

This expression for the pressure gradient is obtained also from the direct solution of the problem for the stabilized flow of a Newtonian fluid. Thus, if for the given formulation the limiting transition is carried out not on the level of the initial problem (14)-(15), but after obtaining the solution, then we can obtain the correct

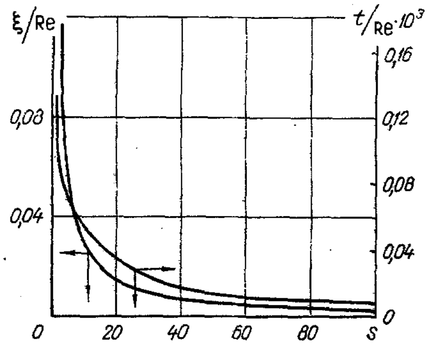

Fig. 3. Dependence of the length of the initial section and the time for achieving a steady-state regime on the plasticity parameter s.

solution within the framework of the Shiller formulation and for the region of large ξ. This is connected with the singularity of the perturbations with respect to the parameter s.

Solution of the problem (14)-(15) was carried out by the method of characteristics. The system of ordinary differential equations corresponding to Eq. (14) is

$$\frac{(2+\delta)^2 dt}{3} = \frac{5(2+\delta)^3 d\xi}{39-21\delta} = \frac{d\delta}{\dfrac{6}{(1-\delta)(2+\delta)\,\mathrm{Re}} - \dfrac{s(1-\delta)}{\delta}}. \tag{20}$$

Introduction of the new variables $\omega = (1-\delta)^2$, $s' = s\mathrm{Re}$ enables us to eliminate Re from system (20). Then for the steady-state flow regime, i.e., for the region near the inlet, the thickness of the boundary layer is determined from the solution of the problem (the primes are dropped)

$$\frac{d\omega}{d\xi/\mathrm{Re}} = \frac{10(3-\sqrt{\omega})^3}{18+21\omega}\left(\frac{6}{3-\sqrt{\omega}} - \frac{s\omega}{1-\sqrt{\omega}}\right), \quad \omega(0)=0, \tag{21}$$

and for the regime of pure nonsteady flow (the region far from the inlet)

$$\frac{d\omega}{dt/\mathrm{Re}} = \frac{2(3-\sqrt{\omega})^2}{3}\left(\frac{6}{3-\sqrt{\omega}} - \frac{s\omega}{1-\sqrt{\omega}}\right), \quad \omega(0)=0. \tag{22}$$

Certain results of the numerical calculations are represented in Figs. 2 and 3. An increase in the plasticity parameter s in the stationary regime leads to a sharp decrease in the quantity ω, which equals the square of the width of the zone of shear flow (Fig. 2a). In the regime of nonsteady flow for small time of the process the effect of the plastic properties is insignificant (Fig. 2b). However, in proportion to the increase in the flow the effect of the plastic properties becomes sizable also for a region of the channel sufficiently far from the inlet, i.e., at points where flow that is steady with respect to the longitudinal coordinate is realized.

The effect of the plasticity parameter on the time for achieving a steady-state regime and the length of the initial reaction is determined. In this case, the flow was assumed to be stabilized (with respect to the temporal and spatial variables) when the thickness of the boundary layer $1-\delta=\sqrt{\omega}$ attains 99% of its steady-state value. The intensification of the plastic properties leads to a decrease both in the time for achieving the steady-state regime and also in the value of the initial section, where the rate of decrease is especially significant for small values of s (Fig. 3).

NOTATION

x, y	are the dimensional coordinates;
u, v	are the dimensional velocity components;
p	is the pressure;
ρ	is the density;
μ	is the plastic viscosity;
h	is the half-width of the channel;
δ	is the coordinate of the boundary of the shear zone;

U	is the dimensional velocity of the "quasisolid" core;
τ_0	is the yield stress;
V	is the dimensional velocity at the inlet;
Q	is the flow rate;
ξ, η	are the dimensionless coordinates;
w, φ	are the dimensionless velocity components;
s	is the plasticity parameter;
Re	is the Reynolds number;
W	is the dimensionless velocity of the "quasisolid" core.

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THERMAL INTERACTION BETWEEN GAS LINE AND FROZEN SOILS

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The article examines the numerical solution of the problem of heat exchange during the flow of gas through an underground pipeline taking into account the phase transitions in the soil under various cooling regimes of the gas.

Investigations of the thermal regimes of pipelines running through frozen soil are dealt with in many works [1-6]. These works give most attention to the investigation of the thermal fields of the soil, while the temperature of the pumped medium is taken as constant.

The present work examines the two-dimensional problem of the change of gas along the pipeline and in time, taking into account the dynamics of heat exchange with the environment and of phase transitions in the soil.

The examined problem includes two groups of equations. The first expresses the laws of conservation for a gas moving in the gas pipe, and with the usually adopted assumptions [7], it can change to the form

$$\frac{\beta_0}{w} T_t + T_x = \beta T + \beta_1 \alpha \vartheta |_{r_0} + \beta_3, \quad \frac{1}{w} M_t + M_x = -(\ln w)_x, \quad P_x = -(z_0 R_0)^{-1} \left(\frac{\lambda}{4 r_0} w^2 + g \sin \mu \right) \exp M. \tag{1}$$

We adopt the following boundary conditions:

$$\begin{gathered} T|_{t=0} = T_1(x), \quad T|_{x=0} = T_2(t); \quad P|_{t=0} = P_1(x), \\ P|_{x=0} = P_2(t), \quad 0 \leqslant x \leqslant L, \quad 0 \leqslant t \leqslant t_f. \end{gathered} \tag{2}$$

The equations of the second group describe the distribution of the temperature field of the soil around the pipeline [9, 11]

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